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HIGH POWER CAPABILITIES  
OF  
WAVEGUIDE SYSTEMS

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## ABSTRACT

The high average power capability of waveguide systems is in general limited by thermal considerations. A tolerable waveguide temperature is limited by the corresponding reduction in breakdown threshold. The waveguide temperature as a function of line power is analyzed in detail for convection and radiation cooling. Preliminary breakdown characteristics of a number of high dielectric strength gases are compared and the experimental apparatus for obtaining breakdown data is described. Also discussed are the conditions under which the breakdown power may be scaled or applied to other configurations when non-uniform field distributions are important.

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## PURPOSE

The purpose of this program is to study the various effects in waveguide systems which lead to failures at ultra-high power levels where high average as well as high peak power become limiting factors. Ultimately a handbook will be prepared to serve as a guide for systems engineers. To achieve the objectives of the program a survey is being made of organizations using ultra-high power, further analysis and theory is being carried out, and experiments are being performed.



## INTRODUCTION

There are several factors which govern the waveguide temperature allowed in a high power waveguide system. As was discussed in the previous report one of the prime considerations is the lowering of the breakdown power due to temperature rise either in the waveguide as a whole or in localized heating at such points in the waveguide system as windows, irises, joints or wherever foreign matter has collected. A second consideration is the acceptable loss in the transmission line taking into account the increase in loss due to the increased temperature. A third consideration is the safety of personnel which limits the waveguide temperature to some maximum value.

One aspect of the problem is the means used to remove the heat from the waveguide. Perhaps the simplest and most desirable means for cooling the waveguide is natural convective cooling by the air surrounding the waveguide. This matter will be discussed in more detail; however, by way of introduction a few remarks will be made. First, although the subject of convection cooling is an old problem discussed in numerous texts and handbooks, the theory is not precise and unfortunately the rectangular cross section found in commonly used waveguide is not one which received much attention. A recent example of application of the theory of convection and radiation cooling to waveguide

problems was given by H. E. King<sup>1</sup> where results for horizontal and vertical plates were combined; however, his results do not predict the different rates of cooling which are obtained if the waveguide is rotated 90 degrees. In the calculations of this report for temperature rise versus average power in waveguide it seemed reasonable, at least for very high power waveguide, to consider the contribution from radiation cooling to correspond to that of a black body (i.e., a surface with an emissivity of unity). This is a reasonable assumption to make because it is a trivial matter to paint the waveguide black and thus obtain the full capability of radiation cooling. The convection cooling should not be altered significantly by the presence of the paint.

In examining the problem of temperature rise and breakdown in waveguides the effects of surface roughness and skin effect were also examined and it was generally concluded that this problem becomes important at frequencies above 10 kmc. A review of the characteristics of gases suitable for use in waveguides was performed. This information is being sorted out for presentation in a form that will readily serve as an aid to systems engineer in trying to predict attenuation, temperature rise and deterioration of the breakdown power in waveguides.

DETAILED FACTUAL INFORMATION

A. Limits on Temperature Rise

The acceptable temperature rise in a waveguide is governed by a number of factors. Among them is the increase in waveguide attenuation. Consider a radar system using a common antenna for both transmitter and receiver and assume that the attenuation in the transmission line is primarily due to the waveguide and further assume that the transmission line already is at the limit of permissible attenuation. To arrive at a limit on temperature rise consider that an increase in attenuation of 50 percent is allowed. To a fair approximation the attenuation is proportional to the square root of the temperature (See Tables I and II). Thus, this increase in attenuation corresponds to an increase in the temperature of 2.25 to 1 on an absolute scale or from 20°C to 367°C. Such a temperature rise is so large that other limitations may become more important. Another consideration is a lowering of the power threshold for breakdown. One might, for example, allow only a 25% reduction in the breakdown power with temperature rise as a practical limit for a waveguide system. Since the power is inversely proportional to the square of the temperature of the gas in the waveguide the allowable temperature rise would be from 20°C to 65°C. This value of temperature rise is rather restrictive; but it is compatible with, say, a safety factor of two which is

commonly used in designing waveguide systems. A third restriction on the temperature rise of a waveguide concerns the safety of personnel working on the waveguide or passing through regions where the waveguide is situated. A permissible value of temperature rise in this consideration is very difficult to specify because of the variety of factors including precautions such as hand grips and guard railings or heat shields. In many instances an 80°C temperature rise may be acceptable; but, on the other hand with some protective measures one should be able to allow temperature rises up to 100°C or even to 150°C if peak power will permit.

In view of the above discussion it appears that the most restrictive condition on the temperature rise is that of the lowering of the breakdown power of the waveguide. Various standard techniques for cooling the waveguide are being studied (see Section D) as well as practical methods for minimizing the attenuation in the waveguide.

#### B. Skin Effect and Surface Phenomena

A number of studies have been published on skin effect at microwave frequencies covering the range from perhaps 100 Mc on up to 50 or 60 thousand megacycles.<sup>3,3,4</sup> The major effect has been attributed to surface roughness which becomes important as the skin depth approaches the size of the roughness found on the waveguide surfaces. This effect has been considered from the view that for small values of skin depth the currents must flow up and down over

the hills and valleys of the surface thus the length of the surface has been increased and, effectively, the surface resistance becomes increased. In the references cited it was concluded that for frequencies up to X-band the problems associated with surface roughness are not significant. However, at frequencies above X-band the problems of surface roughness becomes increasingly important and it is not uncommon to find values of attenuation perhaps twice that found for values calculated from the d.c. conductivity of the materials. Also, one of the references<sup>8</sup> pointed out that in trying to establish a connection between the waveguide attenuation and the d.c. conductivity it was very important to measure the actual d.c. conductivity of the material being used in the waveguide instead of using values commonly tabulated. Still another consideration is the fact that in drawing out waveguide the inner surfaces of the waveguide may be work-hardened. If the skin depth is comparable to this layer of work hardened material, then the attenuation again would not agree with values computed using bulk d.c. conductivity.

A contributing factor to surface roughness in the skin effect problem, namely the effective increase in the length that the currents have to travel on the walls of the waveguide, is the effect of corrosion and rough handling of the waveguide. The corrosion of such materials as aluminum, copper and silver generally occurs in a non-uniform manner resulting in the surface becoming pitted.

Problems in corrosion are particularly critical for waveguides which might be exposed to corrosive atmospheres such as sea air or for that matter the products of some high dielectric strength gases which may decompose into corrosive products. The net effect of this increased attenuation, of course, is a corresponding temperature rise in the waveguide with the possibility of exceeding the temperature rise restrictions discussed above. This information on the skin effect, is being translated into permissible temperature rises for various waveguide sizes.

C. High Dielectric Strength Gases

A search of the literature on breakdown phenomena discloses that a number of gases have been used over the years for their good dielectric strength. Among these gases are  $\text{SF}_6$ , freon, nitrogen, as well as mixtures of such gases, high pressure air and finally completely evacuated waveguide. Our attention so far has been directed towards high pressure gases with properties similar to  $\text{SF}_6$  and freon gases. It has been published that improvements in power handling capabilities may be as large as 10 to 1 for both  $\text{SF}_6$  and some freon gases<sup>5</sup>. A possible advantage of some of the freon gases appears to be that they are less costly than  $\text{SF}_6$ .

The relative dielectric strength of several gases are presented in Figure 1 showing the d.c. breakdown characteristics which were all taken with the same geometry. These results,

published by the duPont Company, were obtained with 2 inch spheres spaced 2 millimeters apart.<sup>6</sup> The breakdown fields as a function of pressure are shown in Figure 1 for the following gases:  $\text{SF}_6$ , three types of freon gases, nitrogen and a curve for air which was derived from the r.f. breakdown data given in Gould's handbook.<sup>7</sup> It is seen that nitrogen and air have about the same characteristics while  $\text{SF}_6$  and freon 115 have an increased breakdown field strengths of about 2.6. The particularly interesting curve is that for freon C318 which has a breakdown field of approximately three times that of nitrogen and air. Translating this information into terms of waveguide power, since the power is proportional to the field strength squared, ratios are obtained for the breakdown powers for the various gases of Figure 1 with respect to air of 10, 7.3, 6.7, 3.9, and 1.22 respectively in decending order.

In order to examine the characteristic of these gases more closely it is instructive to normalize the data as shown in Figure 2 in terms of  $E/p$  as a function of  $pd$ , where  $p$  is the pressure expressed in millimeters of mercury and  $d$  is the spacing in centimeters. In plots of this nature large values of  $pd$  generally correspond to situations in which the breakdown is not governed by the diffusion of electrons to the container walls but by the attachment of electrons to molecules. It should be recalled that the increase in breakdown power in gases such as  $\text{SF}_6$  and freon gases is attributed to the large affinity which these molecules have for

electrons. In such a plot, for large values of  $pd$  the value of  $E/p$  should be constant as illustrated by the r.f. breakdown curve for air which was taken from Gould's handbook.<sup>7</sup> It is seen that the d.c. breakdown data as taken with spherical gaps do not show a range of constant value; rather as the value of  $pd$  increases the d.c. curves tend towards some lower limit thus indicating a continued dependence on  $pd$ . The lack of a constant value in Figure 2 for the d.c. data can be attributed to the fact that the fields may not have been, for that matter should not be, sufficiently uniform in the region between the spherical balls. Thus diffusion exists, i.e., the electrons which are formed at breakdown diffuse out of the regions of higher electric field, even though the process of electron removal is attachment rather than diffusion to the metal surfaces.<sup>8</sup> One of the purposes of the experiments described in this report is to obtain a consistent set of data for these non-uniform electric field conditions at r.f. frequencies rather than d.c. conditions where there is a possibility of other phenomena taking place.

From a practical point of view it should be pointed out that some of the freon gases have relatively high boiling points which means that the gas could freeze out at, say,  $-40^{\circ}\text{C}$  or that equilibrium with the liquid state is reached at pressures which are not exceptionally high. For example, freon C318 is in equilibrium with its liquid state at normal room temperatures at a pressure



of about three atmospheres absolute pressure while its vapor pressure at  $-6^{\circ}\text{C}$  is only one atmosphere. Techniques for using such gas in waveguide systems is a subject of another program at Microwave Associates; however, because of the high dielectric strength, it is valuable in this work to point out its characteristics.

D. Further Calculations On Theoretical Limits of Commonly Used Components and Materials

In analyzing the temperature rise in various waveguide components it is recommended that the problem be separated into two relatively independent parts: first, the ability of the waveguide to dissipate heat to appropriate heat sinks and second the attenuation of microwave power in the waveguide which gives rise to the heat which must be dissipated. The reason for dividing the total problem into these two parts is that the attenuation depends on the nature of the inside waveguide surface and may vary considerably depending upon the material, surface conditions, etc. as has been discussed; whereas, the heat dissipation depends on the conditions of the outside waveguide surface.

The waveguide will normally dissipate heat by convection and by radiation. In the particular problem analyzed here it will be assumed that the most efficient cooling processes exist; therefore, it will be assumed that the emissivity is unity for the heat lost by radiation. In practice almost unity emissivity can easily be

accomplished by painting the waveguide black. In another analysis an author was satisfied to use an emissivity of 0.3.<sup>1</sup>

An expression for the total power which can be dissipated from the surface of the waveguide as a function of temperature and waveguide size has been derived and is shown as Eq. (1).

$$P_d = .170(\Delta T)^{5/4} (a+b)^{3/4} L \\ + 10.14 \times 10^{-10} (T_w^4 - T_a^4) (a+b) L \quad (1)$$

In Equation (1) the dimensions of the waveguide  $a$  and  $b$  and length  $L$  are expressed in feet, the temperature is expressed in degrees Rankin and the power is expressed in watts. The first term of Equation (1) is the convection cooling term derived following the suggestion by McAdams<sup>8</sup> that the convection term simply be that for a straight vertical wall, and in place of the height of the wall a length is taken which is the length of path which the air must follow along the surface in cooling it. McAdams in his text pointed out that this way of computing the heat loss by convection cooling gave good results, for example, for circular cross section pipes. It should be pointed out that there is some lack of agreement between the proportionality constants given by McAdams and the Hand-

book of Heating and Ventilation.<sup>9</sup> The second term in the equation is the familiar radiation term with its dependence upon the fourth power of the temperature. Experiments are being carried out with rectangular waveguides which do show general agreement with the theory and empirical values for the proportionality constant are being ascertained. Equation (1) is only applicable for sea level conditions and it is apparent that for the altitudes encountered by high flying aircraft only the term for the radiated power may be applicable.

Application of Equation (1) to the cases of X-band and S-band waveguide is shown in Figure 3 with a plot of waveguide temperature rise as a function of the power which must be dissipated from the outer surface of the waveguide. Two cases are shown for each waveguide size, one for radiation cooling and a second for radiation plus convection cooling. Since the results shown in these curves are independent of the wave propagation on the inside of the waveguide they may be applied in a wide variety of cases without specifying modes of propagation or attenuation in the waveguide. The curves show that at the temperature of boiling water the power dissipation is of the order of 10 to 100 watts per foot of length.

To compute the temperature rise of the waveguide as a function of line power the temperature dependence of the attenuation is required. In Figure 4 attenuation is plotted as a function of temperature for X and S band waveguide made of copper and brass. For

reference purposes, Table I shows the dependence of the d.c. resistivity of several metals upon absolute temperature and further shows that the resistivities for commonly used metals fall between copper and brass. These values may be substituted into the standard equations of attenuation for various waveguide modes. Table II shows attenuation expressions cast into forms which are particularly useful for comparing various modes of propagation. This is essentially a modification of the expressions given in a similar table in the previous report.

The actual temperature rise as a function of line power can now be computed with the aid of Figures 3 and 4 and the expressions for attenuation given in Table II. In this discussion only the lowest mode of propagation in rectangular waveguide will be considered at a frequency near the center of the pass band. In Figure 5 the temperature rise is plotted for standard X band and S band waveguide for radiation cooling alone and for radiation plus convection cooling. Two different materials, copper and brass, were used for each case. To facilitate the calculations discussed above, a graph was plotted of the ratio of the line power to the dissipated power as a function of attenuation of the waveguide in db per foot. This is the simple linear relationship on log paper shown in Figure 6. It is seen in Figure 5 that for a temperature of about 200°F standard X band waveguide can carry between 0.8 and 3.5 kilowatts depending upon the material used

and the effectiveness of cooling, while standard S band waveguide can carry between 15 and 60 kilowatts. Of course, if more effective cooling is desired, it can be had at the expense of cooling fins, air blowers, liquid circulating systems, etc.

A familiar problem to microwave engineers is that of selecting the proper mode of propagation where the major considerations are power handling capability and reduction of attenuation. An instructive way to compare the various modes of propagation in both circular and rectangular waveguide is to calculate attenuation as a function of the ratio between the size of the waveguide and the free space wavelength. For circular waveguide the size would be the diameter of the pipe while for rectangular waveguide one might take one of the dimensions of the waveguide. Since the attenuation equations shown in Table II indicate a dependence on frequency to the  $3/2$  power, whereas the remaining functional variation involves only the dimensionless ratio of the waveguide size to the free space wavelength, it is instructive to examine the attenuation for a fixed frequency while varying the dimensionless ratio by means of the size of the waveguide.

Comparison of various waveguide modes are shown in Figures 7 and 8 where the  $TE_{01}$  mode in circular waveguide is used as a reference. This mode is the unusual mode which can have a very low attenuation relative to all other modes found in waveguide systems. Various lower order modes are indicated in the figures

showing that the  $TE_{01}$  circular mode has remarkable advantage when the ratio of waveguide size to free space wavelength exceeds a factor of about 2. Thus for very short wavelengths it may be deemed practical to make a waveguide which is perhaps ten times the free space wavelength and thereby obtain a very significant reduction in attenuation. On the other hand, at longer wavelengths it may be very impractical to make a waveguide whose size is perhaps ten times the free space wavelength. For example, at S band where the free space wavelength is 10 centimeters, low loss is obtained if one builds a circular waveguide which is a 100 centimeters or almost 3 1/2 feet in diameter. A further important consideration in comparing these various modes of propagation is the ease with which various microwave components such as isolators, duplexers, phase shifters, etc. can be fabricated. Therefore, if one is limited in size to a value of approximately two wavelengths it would definitely be advantageous to use oversized rectangular waveguide in the  $TE_{10}$  mode since one can easily obtain the various microwave components for this mode.

#### E. Analytical Procedures in Breakdown Studies

The purposes of the experiments are twofold. The first object is to run careful tests on various gases which might be used in waveguide because of their high dielectric strength. For results of such measurements to be included in the handbook it is essential that all of these gases be measured under carefully

controlled conditions. Particularly, the degree of non-uniformity of the electric field must be known and controlled and the purity of the gas must be adequate. Other contributing factors for high dielectric strength gases must also be considered before recommendations can be made as to how the gas might be most effectively used. The second purpose of the experiments will be to evaluate the characteristic parameters of breakdown in various waveguide components. Whereas the experiments described above will be run in carefully controlled configurations, so that the field strengths may be independently determined, in waveguide components such as bends and steps, etc., the electric fields will not be known independently. Therefore, the measured results will be used primarily for scaling the results from one waveguide size to another.

Experimental evaluation of the characteristic parameters used for describing breakdown in waveguide elements becomes useful if the results can be scaled from one size waveguide to another. In other words it would be desirable to limit the experimental work to, say, one frequency and know that the results can be applied in some manner to similar configurations at other frequencies. For this reason the breakdown theory will be reviewed and conclusions pertinent to scaling will be drawn.

The usual starting point for breakdown calculations begins with an equation of the form shown in Equation (2):

$$\frac{dn}{dt} = (\nu_i - \nu_a) n + \nabla(\Gamma) \quad (2)$$

where  $\Gamma = -\nabla(Dn)$  is the particle current,  $D$  is the diffusion coefficient, and  $n$  is the electron density. The left hand side of the equation represents the rate of change of the electron density in the region where breakdown is occurring. The first term on the right hand side of the equation is the difference between the frequency of ionization and the frequency of attachment by electrons. These are processes which involve the intrinsic properties of the gas. The second term is a diffusion term which involves in addition the geometric configuration of the region in which breakdown is occurring. In following through the sequence of events leading to a breakdown consider the variation of  $\nu_i$  and  $\nu_a$  in Equation (2) as the power is increased. The quantity which finally dominates the equation is  $\nu_i$  which initially at low electric field strengths is very small and so the term  $\nu_i - \nu_a$  is negative. This means that the rate of change of electron density is negative and so there is no possibility of having a runaway production of electrons leading to a breakdown. On the other hand as the electric field strength is increased,  $\nu_i$  increases very rapidly once a certain threshold is reached and so finally as the threshold for breakdown occurs  $\nu_i$  begins to exceed  $\nu_a$ .



For the purpose of scaling and solving the diffusion problem it is convenient to define a term shown in Equation (3),

$$\psi = Dn \quad (3)$$

the product of electron density and the diffusion coefficient. With this term substituted into Equation (2) the partial differential equation shown in Equation (4) is obtained:

$$\frac{1}{D} \frac{d\psi}{dt} = \frac{v_i - v_a}{D} \psi + \nabla^2 \psi \quad (4)$$

The commonly used breakdown condition for long pulses and CW is that the rate of change of  $\psi$  with respect to time be equal to zero. This represents the threshold for which  $\psi$  and consequently  $n$  will begin to increase exponentially and therefore for any further increase in electric field there results a runaway condition leading to breakdown. Applying the breakdown condition in Equation (4), Equation (5) results,

$$\nabla^2 \psi + \frac{v_i - v_a}{D} \psi = 0, \quad (5)$$

which is solved as an eigen value problem for the electric field strength which give the proper value of  $(v_i - v_a) D_0$ .

It has been very fruitful to draw an analogy between r.f. and d.c. breakdown. For d.c. breakdown expressions for  $v_i$  and  $v_a$  may be written as shown in Equations (6) and (7) involving the d.c. mobility and the electric field.

$$v_i = \alpha v_{dc} = \alpha \mu_{dc} E \quad (6)$$

$$v_a = \beta v_{dc} = \beta \mu_{dc} E \quad (7)$$

Next in Equation (8) the diffusion coefficient is expressed in terms of the same d.c. mobility evaluated at the mean velocity of the electrons.

$$D = \frac{2}{3} \mu_{dc} v_{ave} \quad (8)$$

Substituting Equations (6), (7) and (8) into Equation (5) the result shown in Equation (9) is obtained.

$$\frac{\nabla^2 \psi}{p^2} + \left[ \frac{3}{2} \frac{\alpha/p - \beta/p}{v_{ave}} \left( \frac{E}{p} \right) \right] \psi = 0 \quad (9)$$

The terms are arranged so that each of the individual quantities are primarily functions of the ratio of E to p. This is a particularly useful form for the Equations since it has been established that the ratio of E/p is a significant variable in  $\alpha$  and  $\beta$ . For r.f. breakdown an effective value of E may always be found<sup>7</sup> and the term multiplying  $\psi$  in Equation (9) has been defined as a high frequency ionization coefficient. This term will be expressed by the symbol  $\zeta$  as shown in Equation (10).

$$\frac{\nabla^2 \psi}{p^2} + \zeta \left( \frac{E}{p} \right)^2 \psi = 0 \quad (10)$$

In order to analyze the problem of scaling in geometry, each distance coordinate will be specified as the product of a characteristic length multiplied by a dimensionless variable which is appropriate for the geometry used. For example, in the case of a circular waveguide the characteristic dimension would be the radius of the waveguide while in the case of a small hemispherical bump in a waveguide the characteristic dimension could be the radius of the hemisphere. When the dimensionless variables,  $\xi$ , and characteristic lengths are introduced into Equation (10), the characteristic dimension, S, falls out as a multiplicative factor squared as shown in Equation (11).

$$\nabla^2 \psi' + S^2 p^2 \zeta \left(\frac{E}{p}\right)^2 \psi' = 0 \quad (11)$$

The remaining variable is the ratio  $E/p$ .

The simplest solution of Equation (11) is that for which the variation in  $\psi'$  is negligible so that the remaining term in the equation satisfies the equality only if the high frequency ionization coefficient  $\zeta$  is equal to zero. This is the condition that  $v_i$  equals  $v_a$  or in other words the electron ionization rate is equal to the attachment rate. For various gases there are specific values of  $E/p$  which satisfies this relationship and the values are independent of geometry. These values of  $E/p$ , when used to compute power in a waveguide, bring in a dependence of the breakdown power on waveguide size.

From the viewpoint of waveguide failures the solution of Equation (11) which interests us is that for which spacial variations in  $\psi$  are not negligible and diffusion becomes significant. This was discussed in the preceeding report.<sup>8</sup> Diffusion in the breakdown phenomena may occur even though the diffusion is not to the container walls. All that is required is that a density gradient exist in regions where breakdown is occurring (the diffusion coefficient has been assumed to be constant). In practical waveguide systems this gradient arises from non-uniform electric fields

which give rise to non-uniform rates of electron production. Thus in general diffusion takes place both to regions where electrons become attached to molecules and to the container walls.

Under conditions of non-uniform electric fields the high frequency ionization coefficient,  $\zeta$ , is very sensitive to the values of  $E/p$  and consequently in considering problems of scaling the essential conditions are those for which a particular form of the solution  $\psi$  is applicable. Examination of the previous equations indicate that for a selected maximum value of  $E/p$  in a given geometry there is a family of solutions, i.e., in Equation (11) the variation of high frequency ionization coefficient will remain unchanged in terms of the dimensionless variables. Therefore, a particular solution,  $\psi$ , for fixed excursion of  $E/p$  in a given geometry can be scaled by maintaining the product of the characteristic length and the pressure constant. Note that the characteristic length as defined here is not to be confused with the diffusion length although the two are simply proportional to one another.

A breakdown measurement for a particular geometry whose size is specified by the characteristic dimension  $L$ , can be scaled, for example, to a larger system if there is a corresponding reduction in the pressure but  $E/p$  is held fixed. Consequently as the size of the structure is increased it also follows that the maximum field strength at the breakdown region must be reduced. This result

can be obtained readily by eliminating  $p$  between two equations stating that  $E/p$  equals a constant and  $S \cdot p$  equals a constant. The result is that the product of the electric field times the characteristic dimension is equal to a constant. In scaling breakdown results it is not essential that we know the value of electric field in the region in which breakdown is occurring. It is sufficient to know the field somewhere in the waveguide system, for example, in an unperturbed section of waveguide.

The experimental procedure for evaluating the breakdown parameters pertinent to a particular waveguide geometry which can be scaled to other waveguide sizes will consist of obtaining values of  $E/p$  where  $E$  is the field strength in the unperturbed waveguide remote from the breakdown region. This will be plotted as a function of  $S \cdot p$  where  $S$  is the characteristic dimension of the breakdown region. From the nature of the relationships governing breakdown it is also apparent that each particular geometry must be treated experimentally. In another program at Microwave Associates the analytical solution for this type of problem is being studied.

#### F. Description of Experimental Apparatus

Breakdown experiments will be conducted at C band, 5 kmc, and the results, as discussed above, will then be scaled to other frequencies or waveguide sizes. The cavity which will be used for the

experiments is shown in Figure 9. This cavity operates in the  $TM_{010}$  circular waveguide mode with the discharge taking place between the two ends of the cavity. The spacings between the ends of the cavity is very small compared to the diameter of the cavity in order to ensure a relatively uniform field in the region where breakdown will occur and also to ensure that the diffusion process takes place between the end walls rather than in a lateral direction to the cylindrical wall. This cavity will be used for a precise determination of the parameters of various gas fills. The cavity is coupled to waveguide through coupling holes as shown in Figure 7 and step transitions are required at both input and output ports of the cavity to obtain a good match into the cavity. The reason for having an output port on the cavity was to externally load the cavity so that the Q would not be excessively high and therefore make the tuning and the setting of the frequency of the high power transmitter less critical. Also shown in Figure 9 are the details as to how the cavity can be taken apart so that one of the surfaces can be modified as desired. In principle the information on breakdown obtained in this cavity can then be extended to other geometries providing we have enough additional information about the electrical field distribution and the high frequency ionization coefficient in these other configurations.

A schematic diagram of the circuitry which is used in these measurements is shown in Figure 10. The pulsed power to the cavity

will be varied by a four port isolator which has a variable magnetic field. Breakdown will be detected by a photo cell whose output in turn is fed to a counter. The photo cell looks into the cavity at a bend in the waveguide as shown. A radio active source is placed near the cavity to enhance the probability of breakdown; however, the electron densities produced by the radio active source will not effect the breakdown threshold. As shown in previous work of this type, by L. Gould and others, if one plots the frequency of breakdown as a function of breakdown power, one can extrapolate the results back to the actual threshold for breakdown. Additional apparatus not shown is a pump station which first evacuates the waveguide system and then allows the particular gas under study to be admitted.

In reducing the breakdown results with this cavity, a perturbation technique is used to independently measure the electric field in the cavity as a function of power. These measurements will also be checked against the breakdown in air, the results of which are well known.<sup>6</sup> The second set of experiments will be made with various standard waveguide components. The additional apparatus required for such tests will be a resonant ring, particularly for transmission type components. In order to ensure that breakdown occurs in the test piece and not elsewhere in the system a radio active source will be placed near the test piece. The breakdown will again be observed with the photomultiplier arrangement shown in Figure 10.



### CONCLUSIONS

The most important limitation on temperature rise of the waveguide is the lowering of the breakdown threshold assuming that the waveguide system is operating near its theoretical limits with a safety factor of about 2 in power. The temperature rise of rectangular waveguides, cooled by convection and radiation, was computed for X and S band frequencies considering both copper and brass. From a practical point of view this simple and natural form of cooling is most desirable and it was found that <sup>the</sup> X band  $TE_{10}^{\square}$  rectangular waveguide could handle from 0.80 to 3.5 <sup>kw</sup> kilowatts of average line power while <sup>the</sup> S band waveguide could handle 15 to 60 <sup>kw</sup> kilowatts of average power. Surface roughness or corrosion of the surfaces were not included in the above analysis; as they only contribute to increased losses and consequently increased temperature rise as the frequency increases beyond 10 kmc. Further, ~~as is well known with regard to losses,~~ comparison of the various modes of propagation indicates that the  $TE_{01}^{\square}$  in circular waveguide can be far superior; however, if size is also a consideration, it is shown that this mode offers no advantage over the  $TE_{10}^{\square}$  mode in rectangular waveguide until the size of the waveguide exceeds 2 wavelengths.

Several gases are expected to increase the breakdown power threshold by approximately a factor of ten. This increase however, remains to be verified by the experimental work described. It is

shown, too, that results for a particular geometry with non-uniform fields can be scaled only if both  $E/p$  and  $pS$  are held constant. The ratio  $E/p$  is field strength to pressure and the product  $pS$  is pressure times characteristic length. ↗

PROGRAM FOR NEXT QUARTER

During the next quarter breakdown experiments will be conducted with the C band cavity on various gases and in addition breakdown test will begin on waveguide components. Experiments will also be completed for determining the proportionality constants for convective cooling of waveguides. Calculation on the theoretical limitation will also be continued.

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ILLUSTRATIONS AND TABLES

TABLE I

Table of Resistivity of Common Metals as a Function of Absolute Temperature

Copper	$\rho/\rho_0 = .152 + 1.15 T/T_0$ , $\rho_0 = 1.72 \times 10^{-8}$ ohm meter
Aluminum	$\rho/\rho_0 = -.143 + 1.14 T/T_0$ , $\rho_0 = 2.84 \times 10^{-8}$ ohm meter
Brass	$\rho/\rho_0 = .414 + .586 T/T_0$ , $\rho_0 = 6.74 \times 10^{-8}$ ohm meter
Gold	$\rho/\rho_0 = .0038 + .995 T/T_0$ , $\rho_0 = 2.44 \times 10^{-8}$ ohm meter
Silver	$\rho/\rho_0 = -.113 + 1.11 T/T_0$ , $\rho_0 = 1.64 \times 10^{-8}$ ohm meter

$$\rho_{0 \text{ cu}} = 1.724 \times 10^{-8} \text{ ohm meter at } 20^\circ\text{C}$$

$$T_0 = 293^\circ\text{K}$$

TABLE II

Formula for Waveguide Attenuation\* (cast into Form for Comparing the Different Modes of Propagation)

Rectangular

$$TE_{m0} \quad \alpha = \frac{K}{\sqrt{1 - \frac{m^2}{4\left(\frac{a}{\lambda}\right)^2}}} \left[ \frac{1}{2\frac{b}{\lambda}} + \frac{m^2}{4} \frac{1}{\left(\frac{a}{\lambda}\right)^3} \right]$$

$$TE_{mn} \quad \alpha = \frac{K}{\sqrt{1 - \frac{m^2 + n^2(a/b)^2}{4} \frac{1}{\left(\frac{a}{\lambda}\right)^2}}} \left[ \frac{1}{\frac{b}{\lambda}} \frac{m^2 + n^2(a/b)}{m^2 + n^2(a/b)^2} + \frac{m^2 + n^2(a/b)^3}{4\left(\frac{a}{\lambda}\right)^3} \right]$$

$$TM_{mn} \quad \alpha = \frac{K}{\sqrt{1 - \frac{m^2 + n^2(a/b)^2}{4} \frac{1}{\left(\frac{a}{\lambda}\right)^2}}} \left[ \frac{1}{\frac{a}{\lambda}} \frac{m^2 + n^2(a/b)^3}{m^2 + n^2(a/b)^2} \right]$$

Circular

$$TE_{n\ell} \quad \alpha = \frac{K}{\sqrt{1 - \frac{(p'_{n\ell}/\pi)^2}{\left(\frac{d}{\lambda}\right)^2}}} \left[ \frac{d}{\lambda} \frac{n^2}{p'_{n\ell}^2 - n^2} + \frac{(p'_{n\ell}/\pi)^2}{\left(\frac{d}{\lambda}\right)^3} \right]$$

$$TM_{n\ell} \quad \alpha = \frac{K}{1 - \frac{(p_{n\ell}/\pi)^2}{\left(\frac{d}{\lambda}\right)^2}} \left[ \frac{1}{\frac{d}{\lambda}} \right]$$

where  $K = 3.52 \times 10^{-14} f^{3/2} \sqrt{\rho}$

\* Nepers per meter

FIGURE 1  
DC BREAKDOWN CHARACTERISTICS OF  
HIGH DIELECTRIC STRENGTH GASES

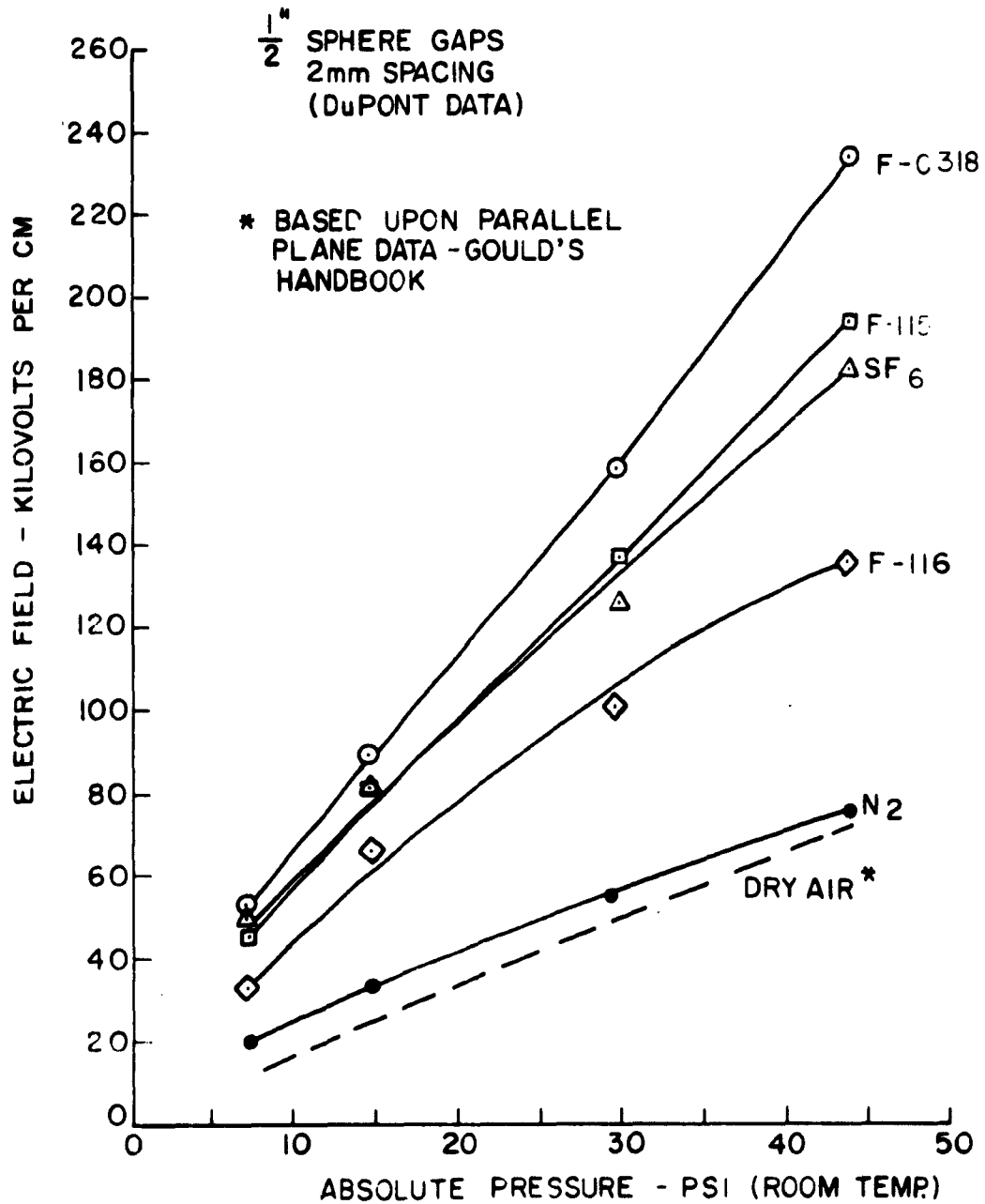




FIGURE 2  
NORMALIZED D.C. BREAKDOWN CHARACTERISTICS  
OF HIGH DIELECTRIC STRENGTH GASES

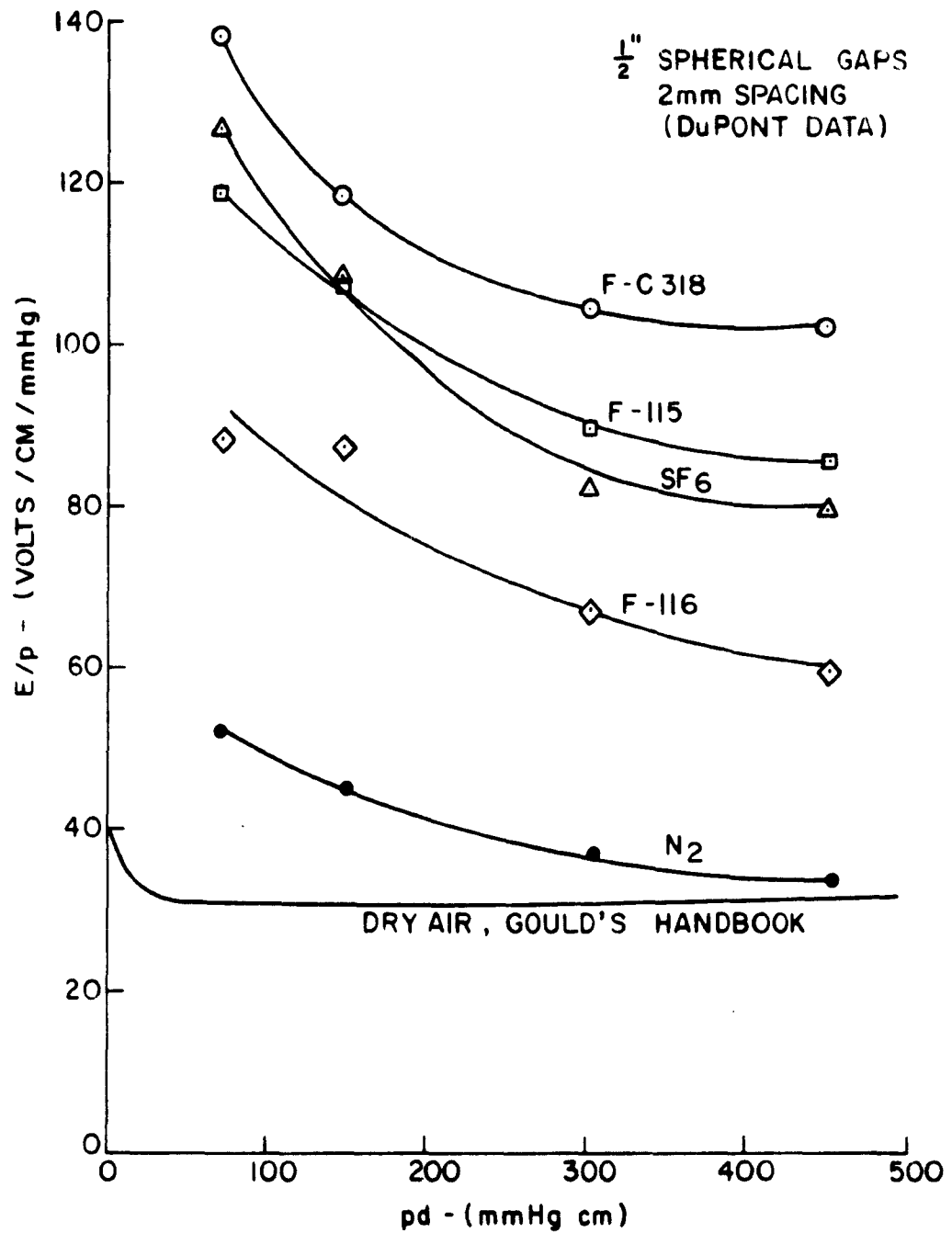


FIGURE 3  
WAVEGUIDE TEMPERATURE RISE  
AS A FUNCTION OF POWER  
DISSIPATION

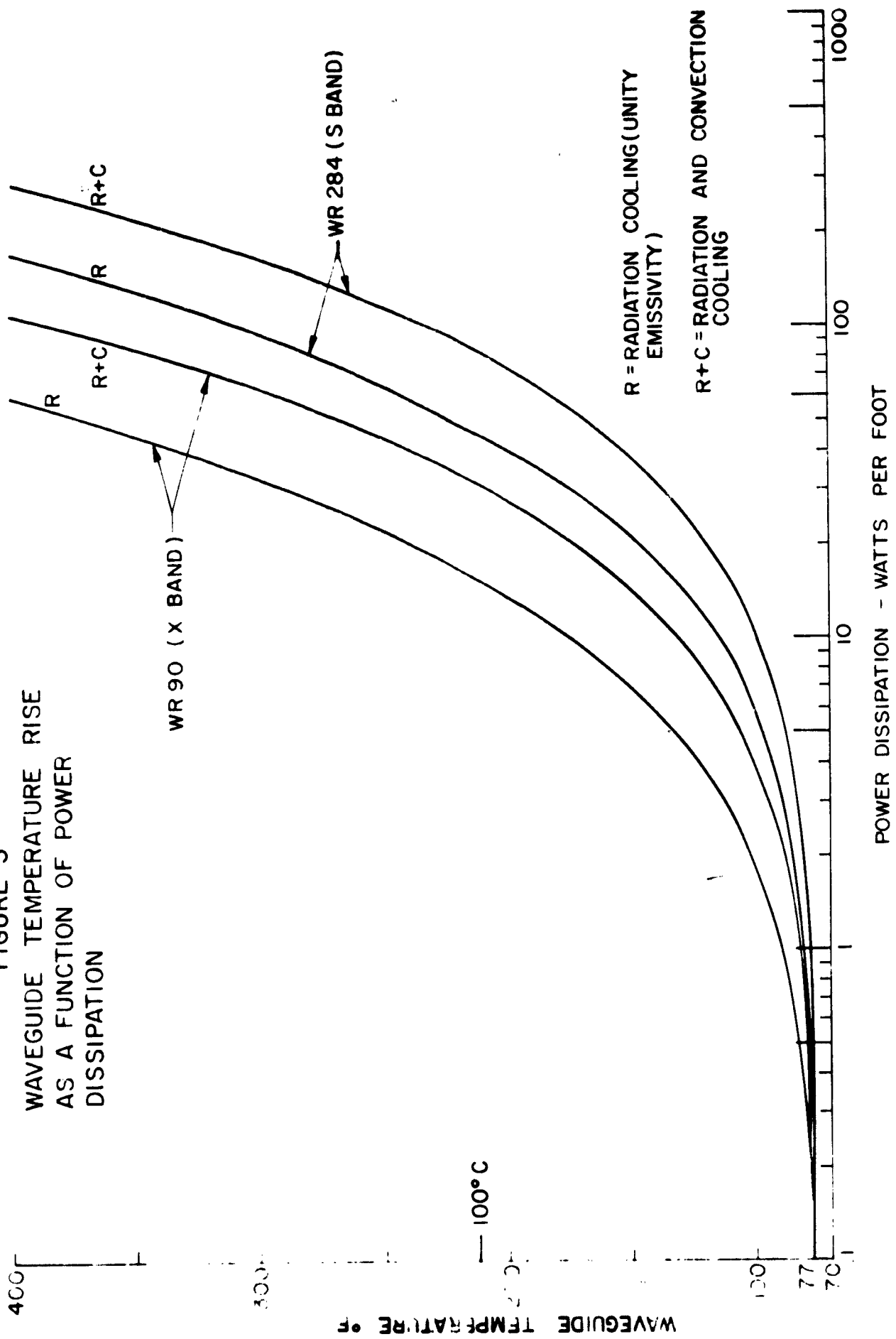


FIGURE 4  
WAVEGUIDE ATTENUATION AS  
A FUNCTION OF TEMPERATURE

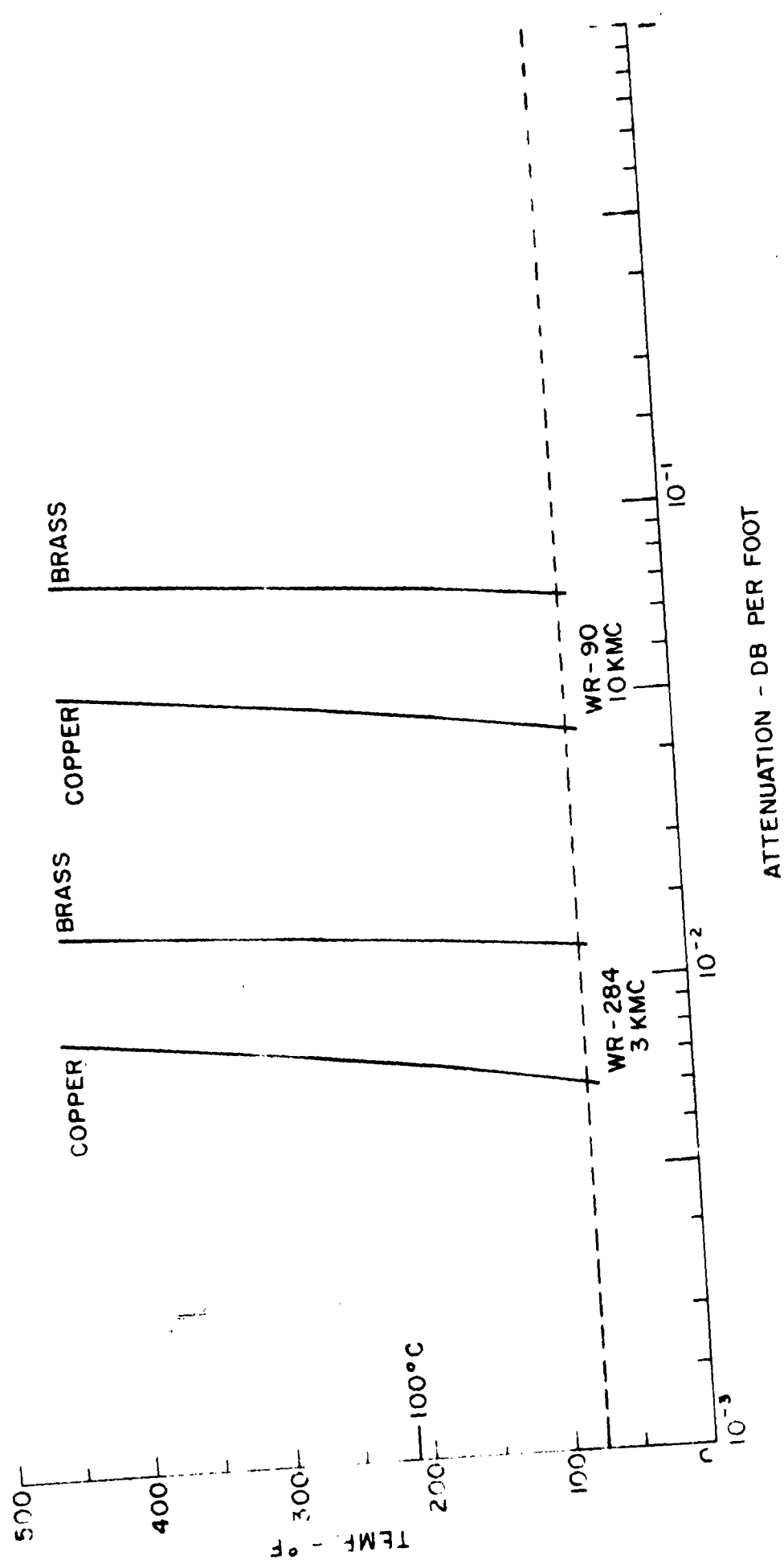


FIGURE 5  
WAVEGUIDE TEMPERATURE RISE  
WITH LINE POWER

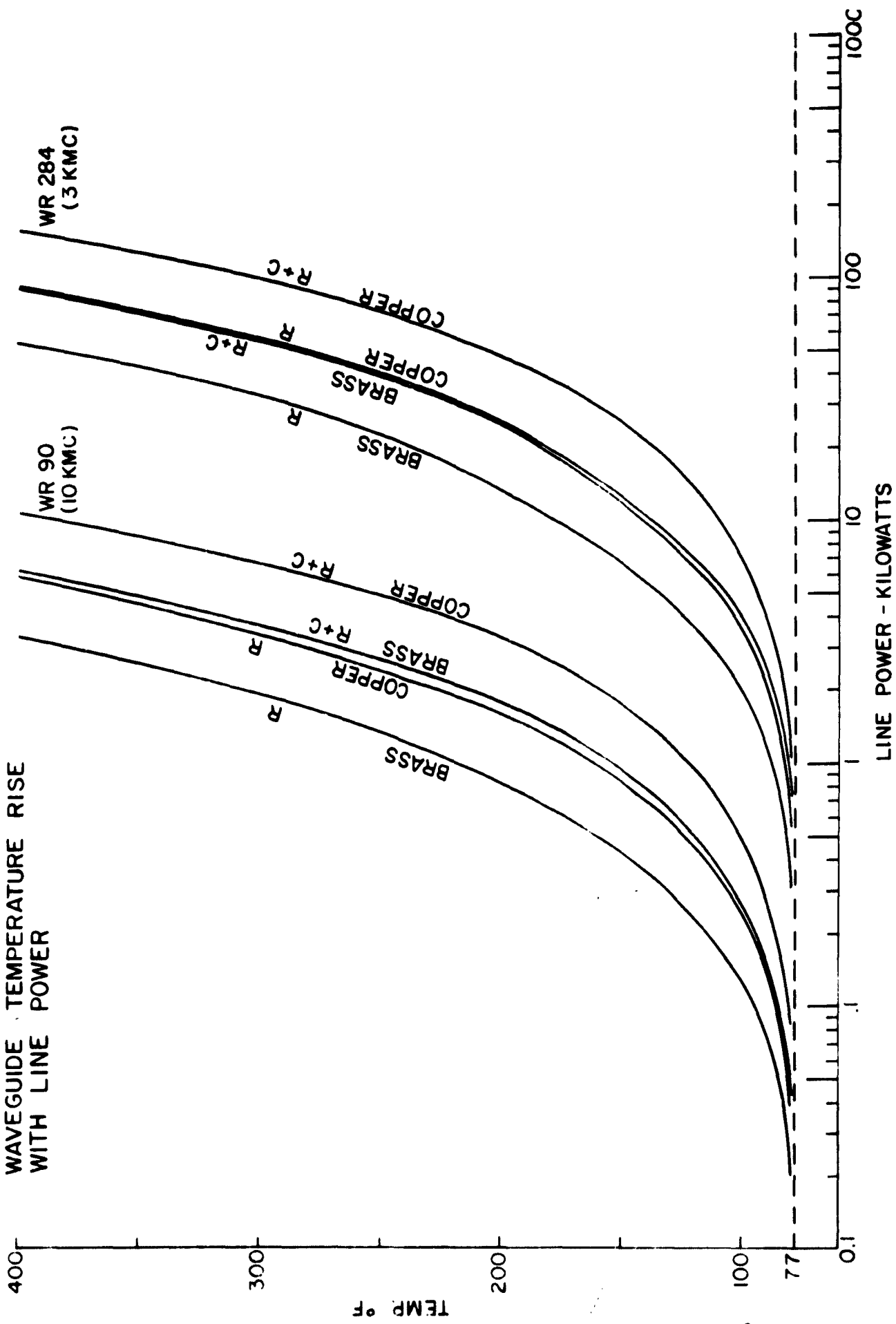


FIGURE 6  
LINE POWER RELATIVE TO DISSIPATED  
POWER AS A FUNCTION OF ATTENUATION

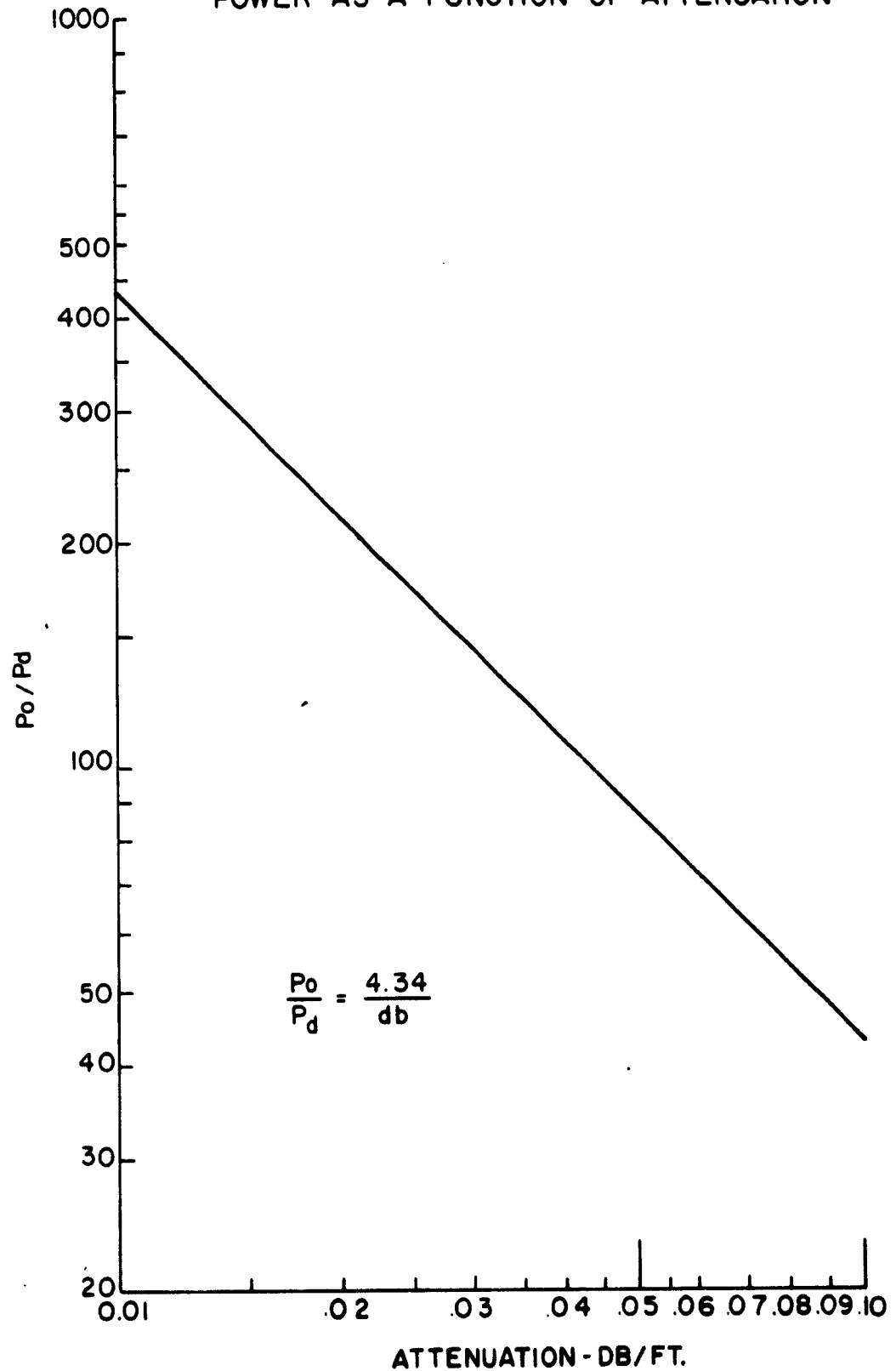
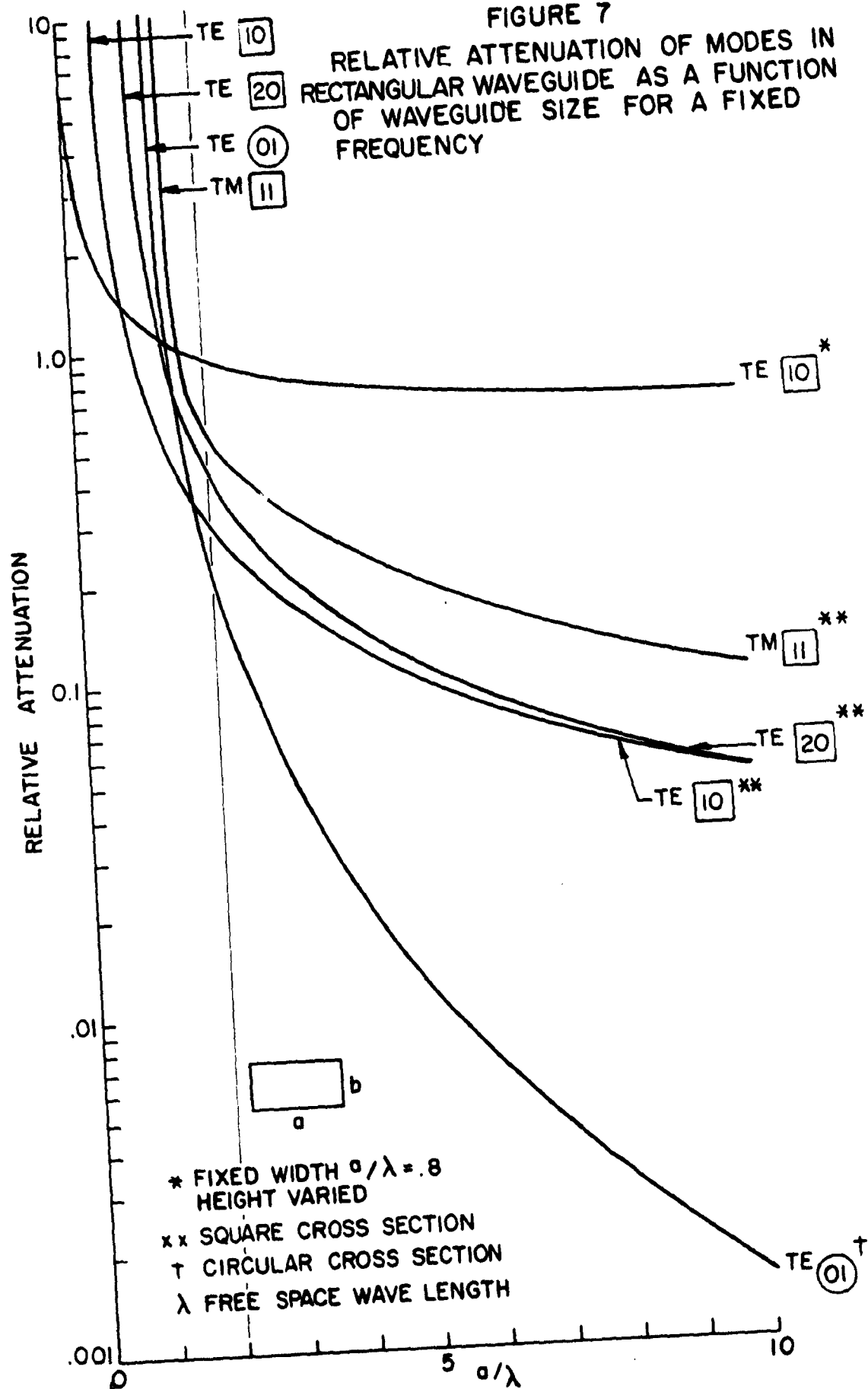
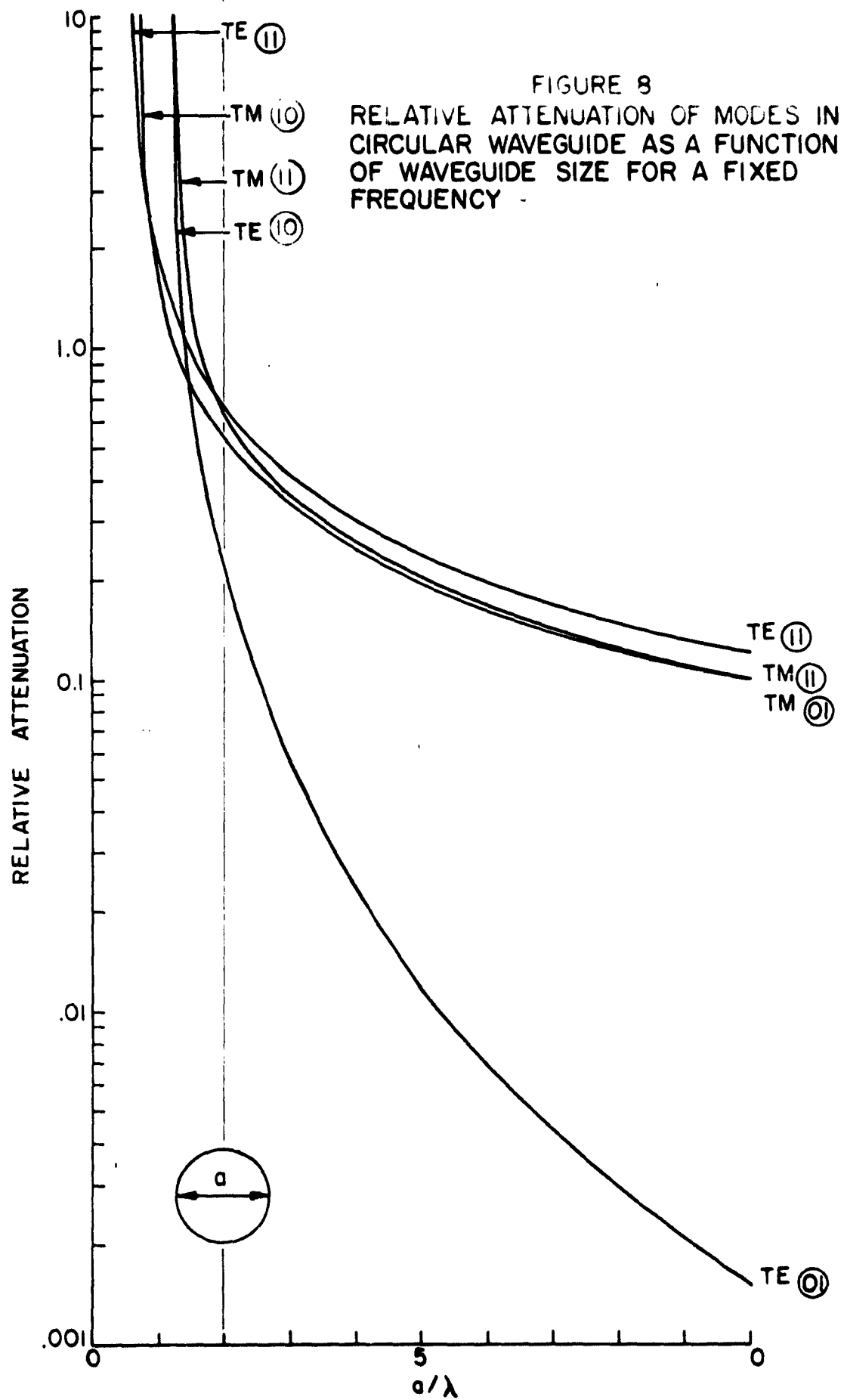


FIGURE 7  
RELATIVE ATTENUATION OF MODES IN  
RECTANGULAR WAVEGUIDE AS A FUNCTION  
OF WAVEGUIDE SIZE FOR A FIXED  
FREQUENCY





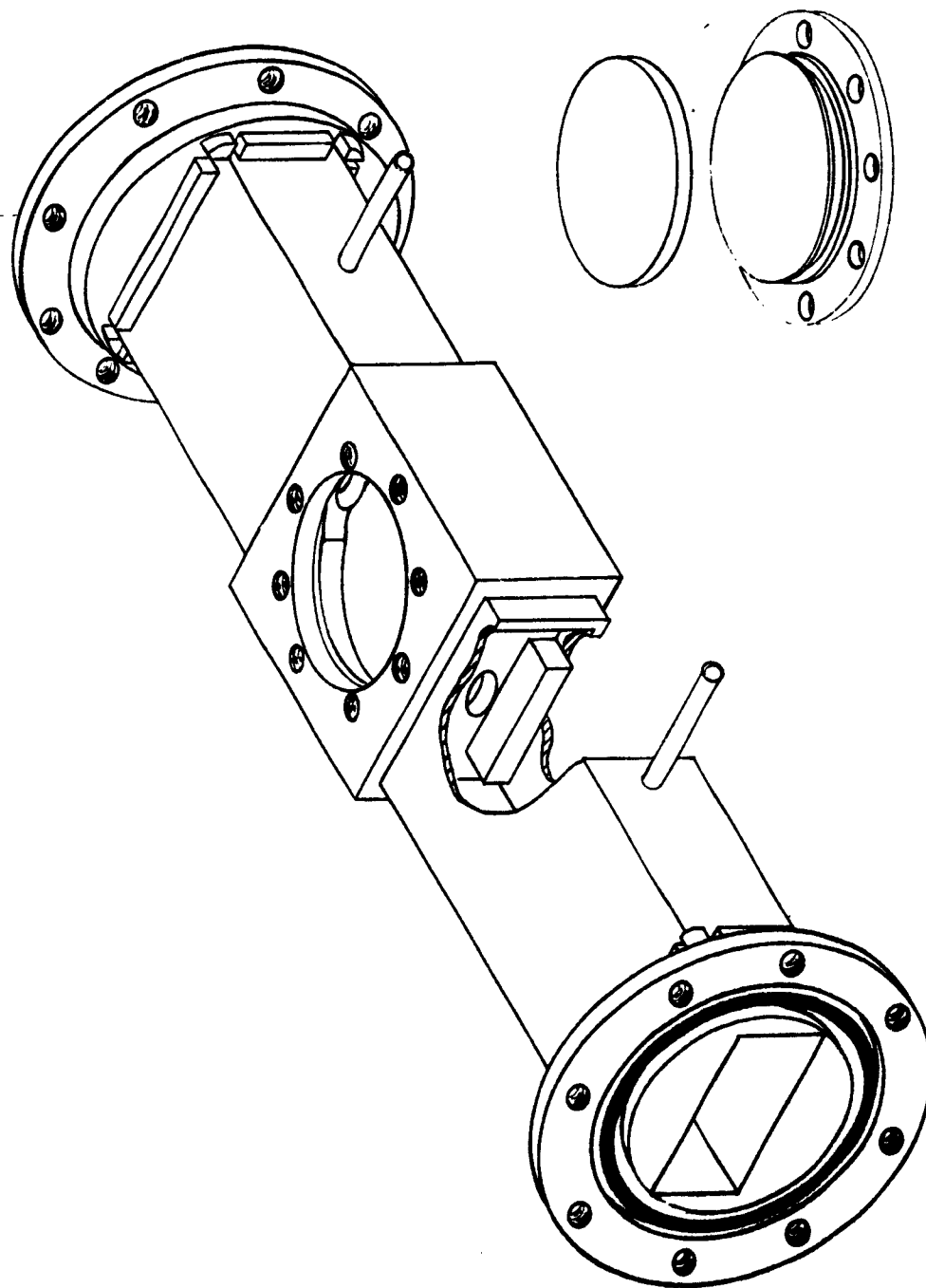


FIGURE 9  
DRAWING OF CAVITY FOR BREAKDOWN TESTS



FIGURE 10  
CIRCUIT DIAGRAM OF BREAKDOWN APPARATUS

